Outline of the lecture

1. Grayscale images

2. Operators on grayscale images

Graph-based image processing

— Grayscale images —

(Professor version)

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Images

Definition

- *Let $\mathbb{V}$ be a set of values*
- *An image (on E with values in $\mathbb{V}$) is a map I from E into $\mathbb{V}$*
- *$I(x)$ is called the value of the point (pixel) x for I*

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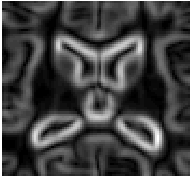
Example

- *Images with values in $\mathbb{R}^+$: euclidean distance map D_X to a set $X \in \mathcal{P}(E)$*
- *Images with values in $\mathbb{Z}^+$: distance map D_X for a geodesic distance in a uniform network*

Grayscale images

- We denote by $\mathcal{I}$ the set of all images with integer values on E
- An image in $\mathcal{I}$ is also called grayscale (or graylevel) image



Grayscale images

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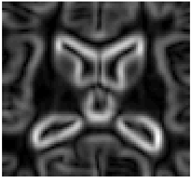
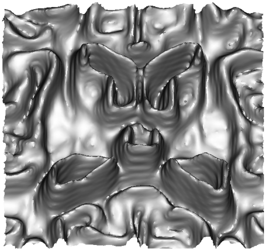
- An image in $\mathcal{I}$ is also called grayscale (or graylevel) image
- We denote by I an arbitrary image in $\mathcal{I}$
- The value $I(x)$ of a point $x \in E$ is also called the gray level of x, or the gray intensity at x



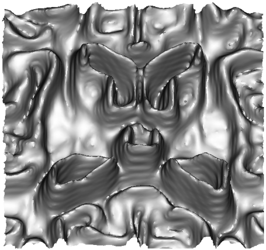
Topographical interpretation

- An grayscale image I can be seen as a topographical relief
  - $I(x)$ is called the altitude of x



Topographical interpretation

- An grayscale image I can be seen as a topographical relief
  - $I(x)$ is called the altitude of x
  - Bright regions: mountains, crests, hills
  - Dark regions: bassins, valleys



Level set

Definition

- *Let* $k \in \mathbb{Z}$
- *The* ***k-level set (or k-section, or k-threshold) of I****, denoted by* I_k*, is the subset of* E *defined by:*
  - $I_k = \{x \in E \mid I(x) \geq k\}$

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I

I_{80}

I_{150}

I_{220}

Reconstruction

Property

- $\forall k, k' \in \mathbb{Z},\ k' > k \implies I_{k'} \subseteq I_k$
- $I(x) = \max\{k \in \mathbb{Z} \mid x \in I_k\}$

Programa de Pós-graduação em INFORMÁTICA

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IM SCIENCE

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Grayscale operators

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Definition (flat operators)

- *Let γ be an increasing operator on E*
- *The **stack operator induced by γ** is the operator on $\mathcal{I}$, also denoted by γ, defined by:*
  - $\forall I \in \mathcal{I}, \forall k \in \mathbb{Z}, [\gamma(I)]_k = \gamma(I_k)$

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Exercice. Show that a same construction cannot be used to derive an operator on $\mathcal{I}$ from an operator on E that is not increasing.

Characterisation of grayscale operators

Property

- *Let γ be an increasing operator on E*
- $[\gamma(I)](x) = \max\{k \in \mathbb{Z} \mid x \in \gamma(I_k)\}$

MM: basic operators

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	0	0	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

Erosion

0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	0	0
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0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

Dilation

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

Opening

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
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0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

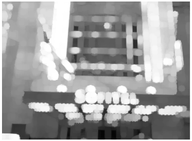
Closing

I

I_{80}

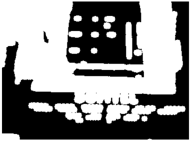
I_{150}

I_{220}



$\delta_\Gamma(I)$

$\delta_\Gamma(I)_{80}$



$\delta_\Gamma(I)_{150}$

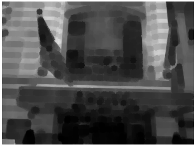
$\delta_\Gamma(I)_{220}$

I

I_{80}

I_{150}

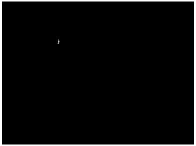
I_{220}



$\epsilon_{\Gamma}(I)$

$\epsilon_{\Gamma}(I)_{80}$

$\epsilon_{\Gamma}(I)_{150}$



$\epsilon_{\Gamma}(I)_{220}$

Illustration: opening on $\mathcal{I}$ by Γ

I

I_{80}

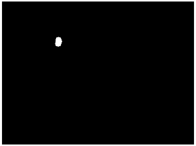
I_{150}

I_{220}

$\gamma_\Gamma(I)$

$\gamma_\Gamma(I)_{80}$

$\gamma_\Gamma(I)_{150}$



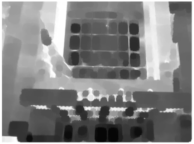
$\gamma_\Gamma(I)_{220}$

I

I_{80}

I_{150}

I_{220}



$\phi_\Gamma(I)$

$\phi_\Gamma(I)_{80}$

$\phi_\Gamma(I)_{150}$

$\phi_\Gamma(I)_{220}$

Dilation/Erosion by a structuring element: characterisation

Property (duality)

- *Let* Γ *be a* *structuring element*
- $\epsilon_\Gamma(I) = -\delta_{\Gamma^{-1}}(-I)$

Dilation/Erosion by a structuring element: characterisation

Property (duality)

- *Let* Γ *be a* ***structuring element***
- $\epsilon_\Gamma(I) = -\delta_{\Gamma^{-1}}(-I)$

Property

- *Let* Γ *be a* ***structuring element***
- $[\delta_\Gamma(I)](x) = \max\{I(y) \mid y \in \Gamma^{-1}(x)\}$
- $[\epsilon_\Gamma(I)](x) = \min\{I(y) \mid y \in \Gamma(x)\}$

Exercise

- Write an algorithm whose data are a graph (E, Γ) and a grayscale image I on E and whose result is the image $I' = \delta_\Gamma(I')$

Acknowledgement

Thanks to the Prof. Jean Cousty at ESIEE/France that gently sent me the slides used in the Morpho, Graph and Image course. Some slides of the Graph-based Image Processing course at PPGINF/PUC Minas under supervision of Prof. Silvio Guimarães will be adapted versions of that course.

- Course - MorphoGraph and Imagery https://perso.esiee.fr/ coustyj/EnglishMorphoGraph/
- Jean Cousty
  - ESIEE Paris, Département Informatique
  - Université Paris-Est, LIGM (UMR CNRS, ESIEE...)
  - E-mail: `j.cousty@esiee.fr`